A Multiresolution Adaptive Approach for Respiratory Motion Modeling

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Abstract—This paper presents a real-time multiresolution approach for respiratory motion modeling. The proposed methodology utilizes a hybrid approach that combines well-known linear identification methods with a novel Global-Local Orthogonal Mapping (GLO-MAP) network. Further, adaptation laws are derived using the recent advances in adaptive control to adapt for respiration models in real-time from imagery data. Finally, the effectiveness of the proposed technique is illustrated by testing them on patient's chest motion data captured by imagery data of external markers.

I. INTRODUCTION
Radiation therapy has emerged as one of the viable tool in the treatment of all kind of cancer, however; the respiratory motion poses significant challenges for treating tumors in the prostate, breast, thorax or abdomen [1], [2]. It can distort the shape of an object, degrade the anatomic position reproducibility during imaging, and necessitate larger margins during radiation therapy planning. The clinical benefit of radiotherapy in the treatment of cancer must be balanced against the adverse effects of the radiations on the healthy tissue. These adverse effects can include ischemic heart disease, pneumonitis and pulmonary fibrosis, erythema, telangiectasia and ulceration of the skin, and bone necrosis in the ribs and sternum [3], [4]. Most of these adverse effects can be related to the patient’s internal organ motion and other deformations mainly caused by setup errors and the respiratory motion which occurs during the duration of the treatment [5], [6]. For example, if there is internal organ movement during the treatment, the delivered dose of radiation may not add up to the desired total dose. Moreover, it is conceivable that a part of the tumor may not receive primary radiation at all because it is always hidden. Recently, Yu et al. [7] have shown that consideration of the interplay between internal organ motion and radiation therapy can change the intensity of the delivered raditions by as much as ±50% from the planned dose value without considering internal organ motion. In short, internal organ motion can distort the shape of the tumor, degrade the anatomic position reproducibility during imaging, and necessitate larger margins during radiation therapy planning. It also causes inaccuracy in estimating the tumor volume, thereby preventing an effective dose escalation for the treatment of a target tumor. It is therefore mandatory that the motion of internal organs should be accounted for, and, correlated to periodic characteristics of the breathing process in addition to estimating the setup errors [8]–[13]. Recent studies [14]–[16] have highlighted the importance of reducing the targeted dose of lethal radiations to surrounding structures, in particular heart and lung in case of breast cancer. Attempts to minimize its adverse effects on radiation therapy represent a significant problem in achieving the goal of conformal radiation therapy. Two techniques are being used to track tumor motion: (1) tumor monitoring with implanted internal radiopaque fiducial and fluoroscopic imaging [12] and (2) indirect tumor monitoring with external markers and/or respiratory sensors [9], [13], [16]. The precise 4D location of a moving internal organ can be obtained through real-time X-ray imaging of internal fiducial markers and stereo imaging of external markers during treatments (Fig. 1(a)). Although the tracking system can follow organ motion, any real-time imaging system inevitably has finite time delay. Such a time delay encompasses many treatment system latencies such as image acquisition, image processing, communication delays, control system processing, mechanical damping, etc. During this entire time delay, the internal organ is still in motion, leading to a resultant time lag in the system’s response and hence the treatment as shown in Fig. 1(b). Thus, it is very important that the tracking system should be directed to an estimated position instead of an observed one. The ultimate objective of our research work is to develop safe and effective “Adaptive Conformal Radiation Therapy” for cancer treatment while minimizing the relapse rate of tumor and side effects of the lethal radiation dose. The main aim of this paper is to introduce a novel multiresolution algorithm which can be used to precisely predict the respiratory motion based on the real-time
imagery data from external markers.

The structure of paper is as follows. First, a multiresolution approach is introduced for the respiratory motion modeling followed by an adaptive algorithm to adapt for model parameters in a real-time. Finally, the proposed algorithms are validated by testing them on real-time external marker data.

II. MULTiresOLUTION APPROACH

In this section, we propose a novel hierarchical multifunctional approach for modeling breathing motion for real-time radiation therapy applications. The main research hypothesis is that the overall respiratory motion modeling process can be split into two sequential subprocesses i.e., off-line modeling followed by the adaptive online modeling.

The offline modeling process can be used to choose a mathematical model, based on prior knowledge about the problem. For example, the respiratory motion is known to be highly oscillatory; therefore, harmonic functions with frequencies close to those of the actual breathing can be used in the basis set. These basis functions can also be constructed intelligently by studying local behavior of observed breathing pattern by the use of methods like Principal Component Analysis or Fourier Decomposition. Hence, offline modeling process implicitly defines a transformed state space that is physically motivated to capture the best representation of the respiratory motion of the patient. We elect to retain this offline modeling as the starting point for a perturbation to account for any localization errors in patient setup, patient coughing, erratic movements and uncertainty in breathing models.

Let the mathematical model for respiratory motion be written as:

$$\dot{x}(t) = \Omega x(t) + f(x, t), \quad \Omega = \text{diag} \begin{bmatrix} 0 & 1 \\ \omega_i^2 & 0 \end{bmatrix}$$

where, $x(t) \in \mathbb{R}^6$ represent the chest deflection along three cartesian directions due to the respiratory motion. $\omega_i$ is the natural frequency of the respiratory motion along the $i^{th}$ direction and can be learned offline. $f(.)$ is a vector of nonlinearities which compensate for any irregularities in nominal respiratory motion, localization errors and patient movements during the treatment. The global nature of the continuous map, $f(.)$, can lead to globally-optimal network parameters which adequately minimize the approximation error but not to the desired level. An alternative to global-learning is local-learning based upon a “divide and conquer” strategy. Of course, we also have to face the possible discrepancies between adjacent and overlapping local approximations. To mitigate these discrepancies, the basis functions used to obtain different local approximation can not be independent from each other without introducing discontinuity across the boundary of different local regions. Basically, a significant challenge is the lack of rigorous methods to merge different independent local approximations to obtain a desired order globally continuous approximation. We propose a multiresolution algorithm that guarantees local convergence using an appropriate methodology that may vary over the approximation region, but efficiently blend diverse models into a statistically unbiased global representation.

A. Special Functions to Blend Independently Constructed Local Approximations

In this section, we briefly discuss a weighting function approach to blend locally independent approximations to a piecewise globally continuous function. More details about this approach can be found in Refs. [17]–[19]. Let us assume:

1) $T = \{t_1, \cdots, t_n\}$ is a uniform grid with spacing $h$ and $-\infty < t_i < t_{i+1} < \infty$. Uniform grid is
We define a non-dimensional local coordinate \( \tau_i \in [-1, 1] \) as needed, to fit the local behavior of \( f(t) \) at points \( t_i \in T \).

3) \( \mathcal{W} \) is a set of continuous functions in the interval \([-1, 1] \).

We define a non-dimensional local coordinate \( \tau_i \in [-1, 1] \) as \( \tau_i \hat{=} (t - t_i)/h \), centered on the \( i \)th vertex \( t = t_i \). Given \( \mathcal{F} \) and weighting function \( w(\tau_i) \in \mathcal{W} \), the weighted average approximation is defined as:

\[
\tilde{f}_i(t) = w(\tau_i)f_i(t) + w(\tau_{i+1})f_{i+1}(t),
\]

for \( 0 \leq \tau_i, \tau_{i+1} < 1 \) and \( t \in [t_i, t_{i+1}] \).

The weighting function \( w(\tau_i) \) is used to blend or average the two adjacent preliminary local approximations \( f_i(t) \) and \( f_{i+1}(t) \). The global function is given by the expression,

\[
f(t) = \sum_{i=1}^{n} w(\tau_i)f_i(t), \ t \in (-\infty, \infty), \tau_i \in [-1, 1].
\]

(3)

The preliminary local approximations \( f_i(t) \in \mathcal{F} \) are completely arbitrary, as long as they are smooth and represent the local behavior of \( f(t) \) well. In Refs. [17], [20], it is shown that if the weighting functions of Eq.(2) satisfy the following boundary value problem (Eq. 4), then the weighted average approximation in Eq.(2) form an \( m \)th order continuous globally valid model with complete freedom in the choice of the local approximations in \( \mathcal{F} \).

These conditions characterize the set \( \mathcal{W} \). That is, \( \mathcal{W} = \left\{ w(\tau) : \begin{array}{l} w(0) = 1, \ w(1) = 0, \\
\ w(\tau) = 1 - \tau^2(3 - 2|\tau|)
\end{array} \right\} \)

(4)

where \( w^{(k)} = \frac{d^k w}{d\tau^k} \). If the weighting function is assumed to be a polynomial in the independent variable \( \tau \), then adopting the procedure listed in Ref. [17], the lowest order weight function (for \( m = 1 \)) can be shown to be

\[
w(\tau) = 1 - \tau^2(3 - 2|\tau|)
\]

Observe that by choosing the weighting functions given by Eq.(5), we are guaranteed global piecewise continuity for all possible continuous local approximations in \( \mathcal{F} \). One retains the freedom to vary the degree of the local approximations as needed, to fit the local behavior of \( f(t) \), and rely upon \( w(\tau) \) to enforce continuity across knot points.

B. Adaptation Laws

Hence, the vector of unknown nonlinearities can be written as: \( f(x, t) = C^T \Phi(x) + \epsilon \) where, \( \Phi(.) \) is a finite dimensional vector of local polynomial functions and \( C \in \mathbb{R}^{N \times n} \) is a matrix of corresponding influence coefficients. Using this we can write:

\[
\dot{x}(t) = \Omega x(t) + C^T \Phi(x) + \epsilon
\]

But since the influence coefficient matrix \( C \) is unknown we can write an estimate equation \( \dot{\hat{x}}(t) = \Omega \hat{x}(t) + C^T \Phi(x) \). We can then define \( e(t) = x(t) - \hat{x}(t) \), which leads to the following expression:

\[
\dot{e}(t) = \Omega e(t) + \hat{C}^T \Phi(x) + \epsilon
\]

Motivated by developments in adaptive control [21] [22], we can now find update laws for the unknown parameters as follows:

\[
\dot{\hat{C}} = \Gamma_1 P e \Phi^T(x)
\]

where, \( P = P^T \) is a positive definite matrix, obtained from \( P \Omega + \Omega^T P = -Q \) for a positive definite symmetric \( Q \) and \( \Gamma_1 \) is suitable adaptation gain matrix. The convergence of the tracking residual \( e \) can be proved rigorously as discussed in detail in Refs. (17], [23). For a given level of the error residuals \( e \), we can conclude bounded stability, as long as the approximation error \( \epsilon \) satisfies the following bound:

\[
||e|| \leq \frac{|\lambda_{min}(Q)||e|}{2\|P\|} = e_{ub}
\]

(5)

The above inequality gives us an upper bound on the approximation error \( \epsilon \) to guarantee the bounded stability of system identification error \( \epsilon \). The adaptive laws can be modified if this bound is violated to curb parameter drift as is done in traditional adaptive control algorithms and our earlier work [17], [23].

III. Validation

In this section, we illustrate the effectiveness of the proposed algorithms in modeling the patient chest motion data collected by Prof. Dr.-Ing. Achim Schweikard’s group, at the University of Lübeck. Figs. 2(a) and 2(b) illustrate the high precision tracking capability of the proposed technique, tested on data related to the in-plane and out of plane motion of markers located on the chest of a patient. It is clear from Fig. 2, that the periodic displacement corresponding to the respiration correlated motion is interspersed with significant displacement which reflects the shifting of the supine patient. This clearly illustrates the ability of the technique to compensate for variations in respiratory signatures and setup errors.
REFERENCES


